The Effects of Doppler Spreads in OFDM(A) Mobile Radio Systems

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Abstract

We analyze the effects of Doppler spread in mobile channels on Orthogonal Frequency Division Multiplex (OFDM) systems. This is important, since channel variations during one OFDM symbol cause Inter Sub-Carrier Interference (ICI) in OFDM systems, which degrades the performance, since ICI can be seen as additional near-Gaussian noise. The analysis is outlined, and closed-form results given for numerous important practical Doppler spread encountered in mobile channels. We also show that for the case of asymmetrical Doppler spreads, the frequency correction of the receiver can be adjusted so as to minimize the ICI. To extend our work to OFDM(A) mobile radio systems in the uplink case where many users' contributions to the ICI overlap, we have introduced the concept of an equivalent single-user which causes the total ICI, in order to use the same analytical framework. For some channel examples, we simulated the Doppler spread of this equivalent user. Simulation results confirm our analysis and show the importance of the analytical tool to perform realistic system analysis without having to resort to time consuming time-domain simulations.

1 Introduction

It is well known that OFDM systems ([1, 2]) loose their orthogonality when the channel varies over the OFDM symbol time, i.e. when the Doppler spread is a significant portion of the sub-carrier spacing. Analysis of the effect of a classical Doppler spectrum on the C/I ratio for OFDM has been introduced in [3] and employed in methods to combat ICI [4] [5]. We have found a *closed solution* for an infinite number of sub-carriers and classical (Jakes) Doppler distribution, as well as other cases for the Doppler spread, such as the important one of an asymmetrical two path channel. Also, we are able to apply our analysis to OFDM systems for mobile radio scenarios in the case of uplink and downlink with several users; which is the focus of this paper. Also within this context, we investigate the achievable gain from optimally aligning the local oscillator to the current Doppler spectrum, in order to minimize the ICI level.

The paper is organized as follows: we introduce a suitable model for our OFDM scheme and sketch the basics for the derivation of the ICI and C/I due to arbitrary Doppler spreads. For some relevant models for the Doppler spread we present the analytical results. We also discuss the optimal local oscillator frequency setting for a given Doppler spread in a two path channel model. We finally model the uplink and downlink in a multi-user OFDM mobile radio scheme where different user are assigned to different sub-carriers, and discuss and interpret the simulation results.

2 Derivation of the ICI level

2.1 Model Outline

In our analysis, we have *initially* assumed a channel with a known and fixed number of paths; for example a very simple two path model, where the two paths have a different amplitude and Doppler frequency. The individual ICI components which will additively contribute to the total ICI are obtained through varying all sub-carriers and all paths. We begin by presenting a simple model for the OFDM transmission system which is general enough to represent the interference scenario which results from neighbouring sub-carriers cross-talking onto a particular sub-carrier. This so-called inter sub-carrier interference (ICI), is caused by the time variant mobile radio channel which has a significant Doppler spread (in comparison to the sub-carrier spacing). Note that a common Doppler shift can be viewed as a frequency offset and be corrected [6].

The base-band model is quite simple and is represented in Fig. 1 with $2N+1$ sub-carriers. The pulse-shaping filters $g(t)$ and $h(t)$ are matched, and we assume rectangular (in time) forms for both, where T is the useful OFDM symbol length. After the matched filter, we have the sampler for each sub-carrier, which would be followed by, e.g. the decision unit. We are interested in the level of ICI (and the level of the useful signal) after the decision unit. The multi-path channel model is drawn in Fig. 2. The channel models P paths, each with index k, a Doppler frequency $f_d^{(k)}$, attenuation $e^{j\alpha^{(k)}} \cdot a^{(k)}$, and delay $\tau^{(k)}$.

We shall assume that our useful sub-carrier is the one with index zero. All other sub-carriers will now disturb this sub-carrier, due to the lost orthogonality, resulting from the Doppler spread. We have not specifically excluded the extension where we look at a

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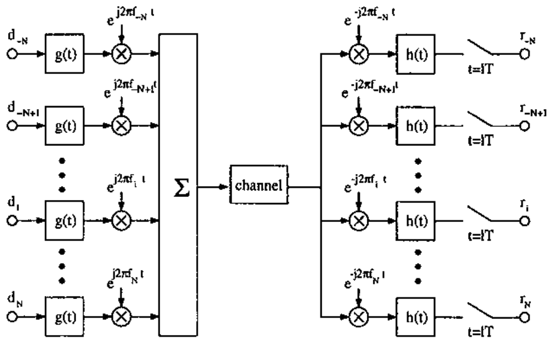

Figure 1: The base-band OFDM transmission model with $2N+1$ sub-carriers. The useful sub-carrier will usually be the center sub-carrier with index $i=0$.

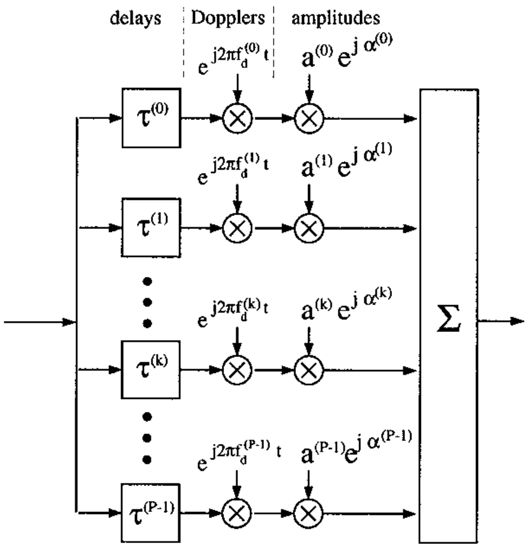

Figure 2: The channel model with P paths, each with a Doppler frequency, attenuation and delay.

useful sub-carrier that lies near the edge of the OFDM spectrum (e.g. at index N or $N-1$), since these sub-carriers will suffer from less ICI. The extension to these cases is straightforward.

Fortunately, the delay of each path is irrelevant to our analysis if all echo durations lie within the guard interval (more correctly referred to as the cyclic extension) employed in OFDM. Observe that the ICI is caused by different paths having different Doppler frequencies, which break the orthogonality between the sub-carriers. Different echo delays $\tau^{(k)}$ and different phases $\alpha^{(k)}$ will essentially result in different overall *phase* rotations in the amplitude leakage from any sub-carrier i, $i \neq 0$, onto the useful sub-carrier with index 0, as far as the contribution via the channel path k is concerned.

The individual ICI components which will additively contribute to the total ICI are obtained through varying all sub-carriers i and all paths k. Because of statistically independent phases for each path, and statistically independent data d_i on each sub-carrier i, all these individual noise components are *uncorrelated.* Therefore, we can sum their energies when computing the ICI energy and are able to ignore the phase rotation of each path, and, of course, the actual phase values of the data symbols transmitted on each sub-carrier. The components are statistically independent when the channel paths' Doppler frequencies and amplitudes are constant; since this is the case for at least the duration of one OFDM symbol, we can use the central limit theorem for large N to model the ICI as zero-mean additive Gaussian noise.

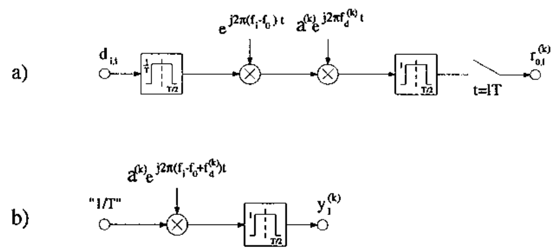

Figure 3: The simplified transmission model: a) assuming only one disturbing sub-carrier i and observing received sub-carrier 0, there is one echo, k. b) Setting $d_{i,l}=1\ \forall l$ allows one to ignore the sampling when determining the variance of the samples $r_{0,l}^{(k)}$.

To proceed, let us simplify the model further. The top half of Fig. 3 shows the data symbol of sub-carrier i for OFDM symbol l, and how it influences the sub-carrier zero, for one echo only, namely with index k. Furthermore, the phases of the channel paths have been set to zero, which we may do for reasons of symmetry. The ICI power in both I and Q components will in reality be the same, but by setting the phases to zero we have forced all the ICI to affect the I component only, whose energy will now be equal to the sum of the ICI energy in the I and Q components in the real system. Thus our analysis pertains to the total ICI energy in I and Q. In order to continue, the model in the top half of Fig. 3 must again be simplified. To determine the variance of the samples $r_{0,l}^{(k)}$ (for a given energy of $d_{i,l}$), the sampling can be ignored if we are able to set $d_{i,l}=1, \forall\, l$. This restriction can be justified since the system is linear and the data of one OFDM symbol l will not affect the output of the receiver for other OFDM symbols $\neq l$. In this case the variance of $y_I^{(k)}$ will equal that of $r_{0,l}^{(k)}$ as long as $E\{d_{i,l}^2\}=1$. This leads to the simple model of Fig. 3 b), where we now need to calculate the variance of $y_I^{(k)}$, which is the integral

$$\sigma^2_{y_I^{(k)}} = \int_{-\infty}^{\infty} \left(\frac{1}{T}\cdot a^{(k)}\right)^2 \delta\left(f-(f_i-f_0+f_d^{(k)})\right) \cdot T^2 \cdot \operatorname{sinc}^2(\pi f T)\, df, \qquad (1)$$

since the spectrum of the signal at the input of the rectangular filter is a Dirac delta with amplitude $\frac{1}{T} \cdot a^{(k)}$, at frequency $f = f_i - f_0 + f_d^{(k)}$, and the power density spectrum (PDS) of the filter is $T^2 \cdot \text{sinc}^2(\pi f T)$. Thus the variance is:

$$\sigma^2_{y_I^{(k)}} = (a^{(k)})^2 \text{sinc}^2 \left(\pi(f_i - f_0 + f_d^{(k)})T\right). \quad (2)$$

2.2 Results for Infinite Sub-Carriers

After proceeding with the analysis (details are shown in [7]), we are able to show that for infinite carriers the ICI component for one path k onto the central carrier is:

$$L_t(k) = (a^{(k)})^2 \cdot \left[1 - \text{sinc}^2(\pi f_d^{(k)})\right]. \quad (3)$$

After further steps which involves integration over the Doppler spread PDFs, and computation of the useful signal component, we obtain closed-form solutions for an infinite number of carriers (albeit resorting to the use of special functions in the solutions). The C/I ratio is computed by dividing the useful signal energy by the ICI energy.

2.2.1 Two-path Discrete Model

For the two-path model the useful signal energy is (regardless of the number of carriers):

$$P_u = (1-p_2)\text{sinc}^2(f_1\pi) + p_2\text{sinc}^2(\Delta f + f_1\pi). (4)$$

The variable f_1 denotes the Doppler frequency of one of the paths with energy $1 - p2$, and p_2 the energy of the second path. The frequency distance between the two paths is denoted by Δf. For an infinite number of sub-carriers, the ICI energy is:

$$N_t = (1 - p_2)\left[1 - \text{sinc}^2(f_1\pi)\right] + p_2\left[1 - \text{sinc}^2(\Delta f + f_1\pi)\right]. \quad (5)$$

2.2.2 Uniform

For a uniform distribution (for scattering environments in three dimensions [8]), we have found the following solutions, beginning with the ICI level:

$$N_t = \frac{\cos(2\pi f_{d_{max}}) + 2f_{d_{max}}\pi\text{Si}(2f_{d_{max}}\pi)}{2\cdot(\pi\cdot f_{d_{max}})^2} + \frac{-1 - 2\cdot(\pi\cdot f_{d_{max}})^2}{2\cdot(\pi\cdot f_{d_{max}})^2}, \quad (6)$$

where Si(.) is the sine integral function [9]. The useful energy is:

$$P_u = -\frac{\cos(2\pi f_{d_{max}}) + 2f_{d_{max}}\pi\text{Si}(2f_{d_{max}}\pi) - 1}{2(\pi\cdot f_{d_{max}})^2}. \quad (7)$$

2.2.3 Classical (Jakes)

The classical Doppler spectrum (see [10]) is actually a long term average. For considerable lengths of time we will often observe an asymmetrical or biased Doppler spectrum, which in practice we can better approximate through our discrete two-path model above, with $p_2 \neq 0.5$. Using the generalized hypergeometric function [9], ${}_pF_q$, where in our case, $p = 1$, $q = 2$, we can write for the ICI level for the classical spectrum:

$$N_t = 1 - {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, 2; -(f_{d_{max}}\pi)^2\right). \quad (8)$$

Similarly, the useful energy is

$$P_u = {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, 2; -(f_{d_{max}}\pi)^2\right). \quad (9)$$

3 Optimal Receiver Frequency Adjustment

We investigate how the receiver should optimally align its local oscillator for a given Doppler spectrum. For small Doppler spreads it is sufficient to effectively set the mean Doppler frequency to zero. For the two path model it is very easy to show that this yields the approximately optimal solution for f_1, namely $f_1 \approx -\Delta f \cdot p_2$.

For large spreads, this is not optimal and results in a loss in C/I of 0.5 to 1 dB. To avoid this loss, the optimal receiver needs to minimize the ICI directly (for example by observing the C/I of some sub-carriers and maximizing the C/I). Such a receiver will per definition find the optimal frequency without having to know the optimal solution for each model for the distribution of the Doppler frequencies. Alternatively, the solution can be found using complex analytical techniques. The question of aligning the local oscillator has repercussions on the performance of the uplink in mobile radio OFDM(A) systems.

4 Uplink Case for OFDM(A)

In a model for the uplink of mobile radio OFDM(A) schemes or MC-FDMA schemes [11], we distinguish two cases: 1) Each mobile user transmits with its nominal carrier frequency; it does not adapt this to its Doppler spread experienced in the downlink, which might be unrelated to the uplink. Also, the basestation receiver does not change its local oscillator frequency for each detected user. This case corresponds to the more simple situation in which the mobile does not necessarily have access to a downlink, or where the downlink suffers from a different Doppler spread (frequency division duplex (FDD), e.g.). The resulting equivalent Doppler frequency PDF is that which is obtained when all users are merged to a single, hypothetical, user. Case 2): Each mobile transmits with a corrected frequency, depending on the

Doppler spread it measures in the downlink: Time division duplex (TDD) is assumed, or a downlink indicating the necessary frequency shift value for the uplink of each user. This is the more suitable case, giving improvements if the short term Doppler spectra of some of the users are asymmetrical. The base station receiver has no need to change the frequency for each user it detects. The individual users minimize their downlink ICI, since they have chosen their carrier frequency accordingly. Since the channel for a user is assumed to be the same in the uplink and the downlink, and the ICI in the uplink is the sum of all users' ICI contributions, the total uplink ICI is also minimized. For a discrete two-path model and four disturbing users we have given an example in Fig. 4 for these two cases. For case 2), the two powerful paths have a lower Doppler frequency due to the frequency correction at each mobile's transmitter, resulting in lower ICI. In Fig. 5 the simulated equivalent user's

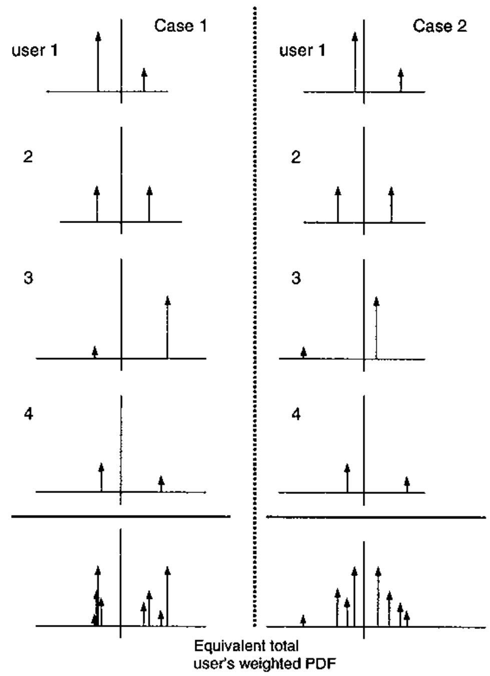

Figure 4: The resulting weighted PDF of the equivalent user for cases 1) and 2). There are 4 uplink users, each of these has a different (shown) two path channel.

PDF according to case 1) without frequency correction and to case 2) with frequency are plotted. It can be observed that with frequency correction the power is located more in the center, resulting in less ICI.

5 Results

The parameters of the analyzed OFDM system (for a mobile radio scenario) are as follows: The bandwidth is $B = 2$ MHz and the carrier frequency is located at $f_c = 2$ GHz. The number of sub-carries for OFDM

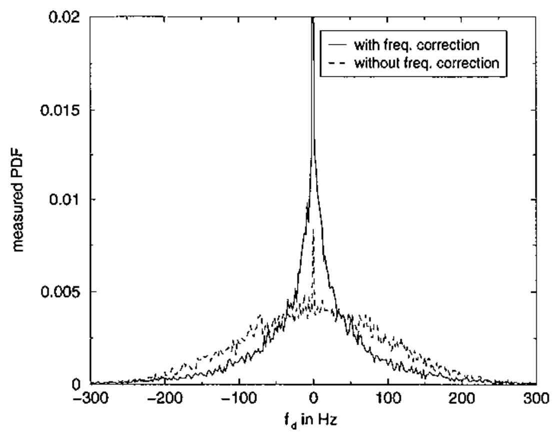

Figure 5: Simulated PDF in the uplink.

is $N = 256$. Thus, the carrier distance is $\Delta f = 7.81$ kHz. We have chosen QPSK for symbol mapping. The maximum number of active users (fully loaded system) is 32, so each user transmits exclusively on its 8 sub-carriers. The 8 sub-carriers of a single user are block interleaved such that they have maximum frequency distance. The data rate per user is 125 kbit/s. The C/I is evaluated separately for the down- and the uplink. For the downlink, we have defined 6 cases with different assumptions about the multi-path propagation in the channel.

We have defined 2 cases based on a two path model with different assumptions on the Doppler spectra: The maximum Doppler frequencies $f_{d_{max},user}$ of the individual users is taken in case a) as the absolute value of a zero-mean Gaussian RV with $\sigma = 0.5 \cdot f_{d_{max}}$, and in case b) from a uniform distribution in $(0, f_{d_{max}})$. The power of p_2 is taken from a uniform distribution in $(0, 1)$ in both cases.

The C/I versus the Doppler frequency is shown in Fig. 6 for the downlink. Analytical and simulation results are shown without frequency correction. Additionally, results are presented with optimal frequency correction for those cases where it is applicable (asymmetric PDFs). As is evident from the figure, the analytical results match perfectly with the simulation results. The C/I versus the Doppler frequency is shown in Fig. 7 for the uplink, and was only evaluated by simulations. The C/I is plotted for the case with and without frequency correction.

6 Discussion of Results

We can observe that for the downlink with uniform Doppler PDF we obtain the highest C/I compared to the other PDFs. With the uniform PDF we have significant contributions at lower Doppler frequencies. Furthermore, the two path model always performs worse (without any frequency correction), even when compared to the classical PDF. Generally, it can be observed that even for very high Doppler frequencies of 600 Hz, corresponding to a velocity of a mobile user

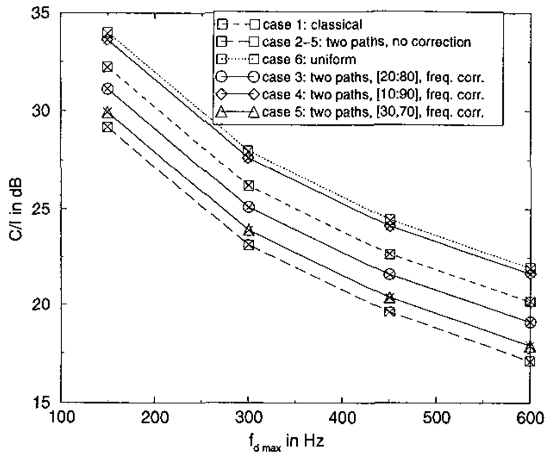

Figure 6: Downlink case. The symbol X shows the analytical results. In parentheses we have shown the path weights in % for the two-path model.

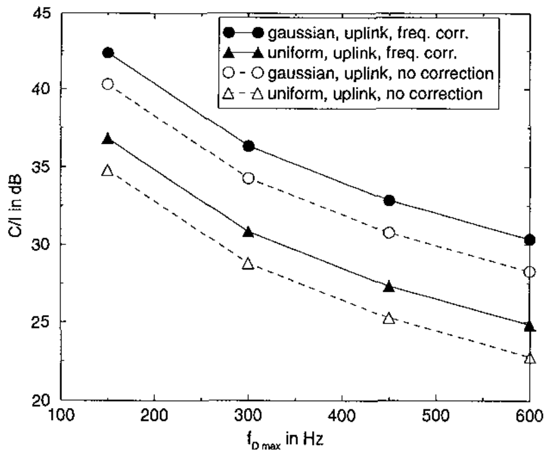

Figure 7: Uplink case. Simulations only.

of 360 km/h, the C/I is greater than 17 dB and can be further improved with frequency correction in the case of the asymmetric two-path model. These C/I values, however, can significantly affect performance, since the ICI can be seen in addition to channel noise. For velocities around 100 km/h the C/I is greater or equal to 30 dB. The results for the uplink show that the more realistic Gaussian Distribution of the maximum Doppler frequency per user outperforms an approach with uniform distribution. The gains due to frequency correction in the uplink are comparable in both cases. Moreover, it can be observed, that the uplink seems to be less critical than the downlink. The reason is that in the downlink we have assumed a worst case scenario where the signals for all users are effected by the same maximum Doppler shift, whereas in the uplink only the fastest users are afflicted with the maximum Doppler spreads. The C/I values which we can obtain from using our techniques are a vital component in exact multi-carrier system designs and analyses, since the ICI effects are not included in typical frequency domain simulations which are standardly used for the physical layer.

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